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On the entry problem for a compressible liquid

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The problem considered is the entry of a thin symmetric wedge impinging normally on the free surface of a compressible inviscid liquid, gravitational effects being neglected. Since the body is thin, the problem is a linear one, and its solution is possible for the whole range of Mach numbers of the body's motion in the liquid. It is shown that the free-surface elevations are lowered considerably as the Mach number increases, and that the presence of the free surface acts so as to lower the pressure differences arising from compressibility.

1. Introduction

The treatment of the entry problem of a body impinging on the free surface of a liquid necessitates dealing with two time-dependent material surfaces, and the problem is thus one particularly difficult to solve. Considerable simplification may be achieved if the motion is geometrically similar at each instant of time (e.g. Cumberbatch 1960). However, even in this case, the inherent difficulty of having non-linear boundary conditions must still be overcome. To avoid this difficulty, Mackie (1962) has considered the impinging body to be thin and has used a linearized theory in the case of a thin wedge entering the liquid surface normally with uniform speed. In two further papers (1963, 1965), he has dealt with the impact of a thin, two-dimensional body of arbitrary shape in normal motion and has evaluated the effects of gravity on the motion.

In the present paper, it is intended to allow for the effect of compressibility of the liquid in the entry problem dealt with by Mackie. In his papers mentioned above, the influence of compressibility is neglected since it will obviously be small for thin pointed bodies unless the impact velocity be large. For example, in the case of water, the speed of small pressure fluctuations is 3200 m.p.h. (at 8 °C) and therefore the entry velocity U of the body must be as great as 640 m.p.h. in order to achieve a Mach number $M = 0{\cdot}2$ for the body's motion in water. It is not difficult to envisage at the present time impacts taking place at this and greater velocities, and therefore it is intended to evaluate the effects of liquid compressibility for what is perhaps the simplest case open to consideration, namely that of a thin symmetric wedge in normal motion, ignoring gravitational effects. Although the body is thin, some of the compressible effects it introduces on striking the liquid are quite evident even for low Mach numbers. For example, it is shown in §4 that for $M = 0{\cdot}2$ the free-stream elevations are lowered on average by about 10 % from those of the incompressible case.

For previous work dealing with the impact of bodies at the surface of a compressible fluid, the reader is referred to papers by Ogilvie (1963) and Skalak &

Feit (1963). These papers deal with the entry problem for a blunt-nosed body in the initial stages of the motion for small depths of penetration. In this case the speed at which the liquid covers the body surface is much greater than the speed of sound in the liquid, and it is found that the effects of compressibility cannot realistically be ignored.

The criterion for neglecting the effects of gravity on the motion is that $gt/U \ll 1$ (Mackie 1963), where t denotes the time elapsed since the vertex of the wedge first touched the free surface. For entry velocities fast enough for compressibility effects in the liquid to have a significant contribution, it would thus appear that gravity terms may be safely ignored for some time after impact.

Finally, in the work which follows, it is assumed that the pressure of the gas in contact with the free surface of the liquid will remain constant. In practice, this is tantamount to assuming that the density of the gas $\rho_g \ll \rho$, the density of the liquid, so that any motion set up in the gas by the moving body does not affect the liquid motion. It is perhaps worth while to examine this question more closely, especially since the Mach number in the gas $M_g \gg M$, the corresponding Mach number in the liquid, thus making the occurrence of shock waves in the gas, with the attendant pressure effects, very likely. In the gas flow about the thin wedge of small semi-vertex angle ϵ the pressure changes present are:

(i) $O(\rho_g U^2\epsilon)$ in linearized theory when M_g is not large and not near unity,
(ii) $O(\rho_g U^2\epsilon^{\frac{2}{3}})$ for M_g near unity, and
(iii) $O(\rho_g U^2\epsilon^2)$ for M_g large in the hypersonic range.

On the other hand, in the liquid, the linearized theory of the present paper yields (see § 5) pressure changes of $O(\rho U^2\epsilon)$ valid for all M less than the hypersonic range of Mach numbers. Thus, pressure effects in the gas are much less than those which arise in the liquid in the various cases, provided that

(i) $\rho_g/\rho \ll 1$,

(ii) $\rho_g/\rho \ll \epsilon^{\frac{1}{3}}$,

and (iii) $\rho_g \epsilon/\rho \ll 1$.

2. Formulation

It is assumed that a compressible liquid is at rest at $t = 0$ and occupies the space $y < 0$ in the (x, y)-plane. The remaining space, $y > 0$, is occupied by a gas. The symmetric wedge moves in the negative y direction with constant speed U and impinges at $t = 0$ on the liquid surface. Since the wedge is thin, having a small semi-vertex angle ϵ, the velocity $\mathbf{q} = (u, v, 0)$ of the liquid is small, so that u/U and v/U are $O(\epsilon)$. The free surface of the liquid is given at time t by $y = \eta(x, t)$, where η is taken in the linearized theory to be small and $O(\epsilon)$, so that the free surface can be approximated by its initial level $y = 0$.

The motion is assumed to be irrotational and described by a potential function $\phi(x, y, t)$, where $\mathbf{q} = \nabla\phi$. ϕ satisfies the linearized potential equation, i.e. the simple wave equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}-\frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2} = 0, \tag{1}$$

where a is the speed at which small disturbances are propagated in the liquid. The linearizing assumptions apply over the whole range of Mach numbers $M\ (= U/a)$ below the hypersonic range, including the sonic region near $M = 1$. No difficulties are encountered near $M = 1$ in the present problem since the region influenced by the body is always finite, so that small disturbances caused

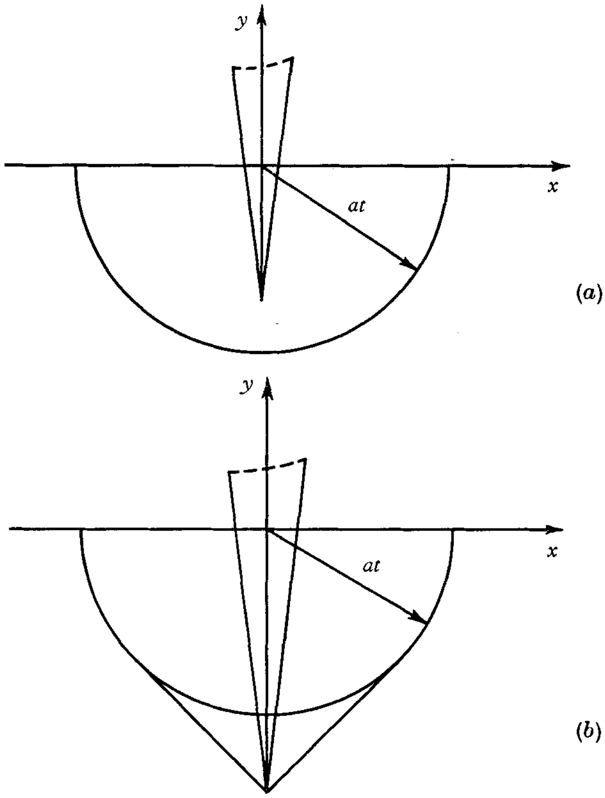

FIGURE 1. Regions influenced by the body at time t. (a) $M < 1$, (b) $M > 1$.

by the passage of the body through the liquid will not build up by superposition as they do for steady flow in an infinite region. The regions of liquid set into motion by the body at time t are shown in figures 1 (a) and (b) for $M < 1$ and $M > 1$, respectively, the depth penetrated in each case being equal to Ut.

Since the motion is symmetrical it is sufficient to consider only the region $x > 0$. The wedge is defined as $x = \epsilon(y + Ut)\,H(y + Ut)$, where $H(y + Ut)$ is the Heaviside unit function. Both the wedge and the free surface are 'material surfaces', and thus the equations to be satisfied on these surfaces are given by

$$\frac{D}{Dt}\{y - \eta(x,t)\} = 0 \quad \text{on} \quad y = \eta(x,t), \tag{2}$$

and

$$\frac{D}{Dt}\{x - \epsilon(y + Ut)\,H(y + Ut)\} = 0 \quad \text{on} \quad x = \epsilon(y + Ut)\,H(y + Ut) \quad \text{for} \quad y < 0, \tag{3}$$

where $D/Dt = \partial/\partial t + \mathbf{q}.\nabla$ denotes the substantive derivative. The boundary conditions (2) and (3) may be replaced by the simple linear equations

$$\frac{\partial\phi}{\partial y} = \frac{\partial\eta}{\partial t} \quad \text{on} \quad y = 0, \tag{4}$$

and
$$\frac{\partial\phi}{\partial x} = \epsilon U H(y+Ut) \quad \text{on} \quad x = 0 \quad \text{for} \quad y < 0, \tag{5}$$

in which only terms of first order in ϵ have been retained.

A third condition to be satisfied is that of constant pressure P_0 on the gas–liquid interface. Bernoulli's equation for an unsteady compressible fluid of density ϕ, pressure P, under no body forces is

$$\int_{P_0}^{P} \frac{dP}{\rho} + \tfrac{1}{2}\mathbf{q}^2 + \frac{\partial\phi}{\partial t} = 0. \tag{6}$$

The arbitrary function of time usually appearing in this equation is absorbed into the potential function ϕ. The assumption is now made that the pressure P in the liquid differs only slightly from the constant value P_0 of the gas. Neglecting the velocity-squared term in (6), which is of $O(\epsilon^2)$, then

$$P - P_0 = -\rho_0 \partial\phi/\partial t, \tag{7}$$

where ρ_0 denotes the density of the undisturbed liquid. The pressure difference $P - P_0$ vanishes on the free surface $y = \eta(x, t)$, and thus (7) leads to the boundary condition

$$\partial\phi/\partial t = 0, \tag{8}$$

applicable on $y = 0$ with no loss of accuracy.

To summarize, the problem is reduced to finding a solution of (1) for $y < 0$, $x > 0$ satisfying the boundary conditions (5) on $x = 0$ and (8) on $y = 0$. Of particular interest are the free surface elevation, which can be determined from (4), and the pressure distribution on the wedge, which can be obtained from (7) as $x \to 0$.

3. The operational solution

Referring to figure 1, it may be seen that ϕ is zero for $x > at$ at a given time t. A Fourier cosine transform may therefore be defined by

$$\bar{\phi}(p, y, t) = (2/\pi)^{\frac{1}{2}} \int_0^\infty \phi(x, y, t) \cos px\, dx, \tag{9}$$

in which the variable x is replaced by the operator p. In addition, the Laplace transform, defined by

$$\tilde{\phi}(x, y, aq) = \int_0^\infty \phi(x, y, t)\, e^{-aqt}\, dt, \tag{10}$$

allows the variable t to be replaced by the operator aq. Applying these transforms simultaneously to the wave equation (1), it follows that

$$\frac{d^2\tilde{\bar{\phi}}}{dy^2} - (p^2+q^2)\,\tilde{\bar{\phi}} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \left(\frac{\partial\tilde{\phi}}{\partial x}\right)_{x=0}. \tag{11}$$

It has been assumed in deducing (11) that $\phi = 0$ and $\partial\phi/\partial t = 0$ at $t = 0$. This latter condition follows from (7), taking $P = P_0$ at $t = 0$. Now from (5), employing a Laplace transform,

$$\left(\frac{\partial\bar{\phi}}{\partial x}\right)_{x=0} = \int_0^\infty \epsilon U H(y+Ut)\, e^{-aqt}\, dt$$
$$= (\epsilon M/q)\, e^{qy/M}. \tag{12}$$

Inserting (12) into the right-hand side of (11), a solution of the differential equation in y is required which will remain finite as $y \to -\infty$. This may be written down directly as

$$\tilde{\bar{\phi}}(p,y,aq) = A(p,q)\exp\{(p^2+q^2)^{\frac{1}{2}}y\} + \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\epsilon M^3}{q}\frac{\exp(qy/M)}{q^2-M^2(p^2+q^2)}. \tag{13}$$

$A(p,q)$ is an arbitrary function which may be determined from the operational form of the boundary condition (8). Using the condition $\phi = 0$ at $t = 0$, (8) yields

$$\tilde{\bar{\phi}} = 0 \quad \text{on} \quad y = 0. \tag{14}$$

On determining $A(p,q)$, the operational form of the velocity potential finally reduces to

$$\tilde{\bar{\phi}}(p,y,aq) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\epsilon M^3}{q}\frac{\exp(qy/M)-\exp\{(p^2+q^2)^{\frac{1}{2}}y\}}{q^2-M^2(p^2+q^2)}. \tag{15}$$

4. The free-surface elevation

Rather than to proceed with the inversion of the operational solution (15), it is more rewarding to consider the simpler problem of determining the free-surface elevation or splash profile represented by $\eta(x,t)$. This approach carries with it the advantage that this profile is easily visualized. To this end, (4) is differentiated with respect to time, and the operational form of $\partial^2\eta/\partial t^2$ is found, using the solution (15). Assuming $\partial\eta/\partial t = 0$ at $t = 0$, then

$$\frac{\partial^2\tilde{\bar{\eta}}}{\partial t^2} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\epsilon U^2}{a\{q+M(p^2+q^2)^{\frac{1}{2}}\}}. \tag{16}$$

It appears that the double operational transform (16) may be inverted directly from the standard tables in three special cases only—namely (i) $M = 0$, (ii) $M = 1$, and (iii) $M \gg 1$. In cases (ii) and (iii), the inversions of the Laplace transforms in q lead to Bessel functions which are very conveniently replaced by simple irrational functions on finally inverting the Fourier cosine transforms in p. The simplification which ensues in these special cases suggests that the general case may be carried out more simply by a direct method. This is in fact the case, and investigation leads to an inversion procedure which could equally well be applied to the more complicated operational solution (15). To illustrate this procedure in the present case a slightly more complicated operational form than that in (16) is chosen. This is given by

$$G(p,q) = \frac{(2/\pi)^{\frac{1}{2}}(\epsilon U^2/a)}{q+M\{q+p(i+\tau)\}^{\frac{1}{2}}\{q-p(i-\tau)\}^{\frac{1}{2}}}, \tag{17}$$

which reduces to the expression in (16) when τ, a real constant, is put equal to zero. The reason for the introduction of this number τ will appear shortly. As will be seen, it allows certain singularities which occur in the Q-plane (defined below) to be displaced in such a way as to make valid a simple inversion with respect to p.

The inversion formulae from the Fourier and Laplace transforms are now employed together with (17) to yield

$$\frac{\partial^2\eta}{\partial t^2} = \lim_{\tau\to 0} -\frac{ia}{2^{\frac{1}{2}}\pi^{\frac{3}{2}}}\int_0^\infty \left\{\int_{\delta-i\infty}^{\delta+i\infty} G(p,q)\, e^{aqt}\, dq\right\} \cos px\, dp, \tag{18}$$

where the integrand in q is analytic in some half-plane $\mathrm{R}(q) \geqslant \delta$. Putting $q = pQ$, then

$$\frac{\partial^2\eta}{\partial t^2} = \lim_{\tau\to 0} -\frac{i\epsilon M^2a^2}{\pi^2}\int_0^\infty \left(\int_{\sigma-i\infty}^{\sigma+i\infty} \frac{\exp(apQt)\, dQ}{[Q+M\{Q+(i+\tau)\}^{\frac{1}{2}}\{Q-(i-\tau)\}^{\frac{1}{2}}]}\right)\cos px\, dp. \tag{19}$$

The path of integration in the complex Q-plane is along the straight line $\mathrm{R}(Q) = \sigma\ (= \delta/p)$, which passes to the right of the singularities of the integrand, The singularities occur at the branch points $Q = \pm i - \tau$, and, for τ small. ($|\tau| \ll |1-M|$), either at the simple pole

$$Q = -M/(1-M^2)^{\frac{1}{2}} + M^2\tau/(1-M^2) + O(\tau^2) \quad \text{if} \quad M < 1,$$

or at the two simple poles

$$Q = \pm iM/(M^2-1)^{\frac{1}{2}} - M^2\tau/(M^2-1) + O(\tau^2) \quad \text{if} \quad M > 1.$$

Thus, provided τ is small and positive, the number σ can be chosen such that $-M < -\tau < \sigma < 0$. This condition makes it possible to arrange for the path of integration in the Q-plane given by $\mathrm{R}(Q) = \sigma$ to pass to the left of the imaginary axis while still retaining the singularities of the integrand on its left. Selecting the path $\mathrm{R}(Q) = \sigma$ in this way, (19) may now be taken a step further, since

$$\begin{aligned}\int_0^\infty \exp(atQp)\cos px\, dp &= \frac{1}{2}\int_0^\infty [\exp\{(atQ+ix)p\} + \exp\{(atQ-ix)p\}]\, dp \\ &= -atQ/(a^2t^2Q^2+x^2),\end{aligned} \tag{20}$$

which is valid if $\mathrm{R}(Q) = \sigma < 0$ for $a > 0, t > 0$. Using (20), assuming that the change in order of integration is justified, (19) becomes

$$\frac{\partial^2\eta}{\partial t^2} = \lim_{\tau\to 0}\frac{i\epsilon M^2a^2}{\pi^2}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{atQ}{a^2t^2Q^2+x^2}\frac{dQ}{Q+M\{Q+(i+\tau)\}^{\frac{1}{2}}\{Q-(i-\tau)\}^{\frac{1}{2}}}. \tag{21}$$

The path of integration $\mathrm{R}(Q) = \sigma < 0$ passes to the right of the singularities given above. However, in addition to these singularities, the integrand in (21) has two new singularities, both simple poles, occurring at the points $Q = \pm ix/at$. The path of integration $\mathrm{R}(Q) = \sigma$ passes to the left of the new singularities, and thus the integral in (21) may be conveniently replaced by the sum of residues of these two simple poles. This may be proved by integrating round the contour bounding a semicircle defined by $|Q-\sigma| < R$, $\mathrm{R}(Q) > \sigma$, containing the points $Q = \pm ix/at$, and then letting $R \to \infty$. On the curved portion of the semicircle, the integrand behaves like $O(R^{-2})$, while the arc length is of $O(R)$ as R becomes large. Thus the integral over the curved portion is of $O(R^{-1})$ and does not contribute as $R \to \infty$. Evaluating the residues at the simple poles $Q = \pm ix/at$, and putting $\tau = 0$, it follows finally that

$$\frac{\partial^2 \eta}{\partial t^2} = \frac{2\epsilon U}{\pi t} \frac{\{1-(x/Ut)^2 M^2\}^{\frac{1}{2}}}{1+(x/Ut)^2(1-M^2)}. \tag{22}$$

(22) may be obtained in an alternative manner, dispensing with the constant τ. This, however, entails deforming the path of integration in the Q-plane on inverting the Laplace transform so that (20) may be validly used. Certain difficulties which arise due to singularities on the imaginary axis have to be overcome. (22) is integrated twice with respect to t, making use of the end conditions $\eta = 0$, $\partial\eta/\partial t = 0$ when $x = at$. The free-surface elevation in terms of $\gamma = x/Ut$ then becomes

$$\frac{\pi}{\epsilon}\frac{\eta}{Ut} = 2\log\left(\frac{1+(1-M^2\gamma^2)^{\frac{1}{2}}}{M\gamma}\right) - 2(1-M^2\gamma^2)^{\frac{1}{2}} + 2\gamma\tan^{-1}\{(1-M^2\gamma^2)^{\frac{1}{2}}/\gamma\} + I, \tag{23}$$

where $$I = \frac{-1}{(1-M^2)^{\frac{1}{2}}}\log\left[\frac{1+(1-M^2)^{\frac{1}{2}}(1-M^2\gamma^2)^{\frac{1}{2}}}{1-(1-M^2)^{\frac{1}{2}}(1-M^2\gamma^2)^{\frac{1}{2}}}\right] \quad \text{if} \quad M \leqslant 1,$$

and $$I = \frac{-2}{(M^2-1)^{\frac{1}{2}}}\tan^{-1}\{(1-M^2\gamma^2)^{\frac{1}{2}}(M^2-1)^{\frac{1}{2}}\} \quad \text{if} \quad M \geqslant 1.$$

The splash profiles are shown in figure 2 for $(\pi/\epsilon)(\eta/Ut)$ against x/Ut for various Mach numbers, including the incompressible case first derived by Mackie (1962). As can be seen, there is no special significance about the result for $M = 1$. In each case the depth penetrated by the wedge is Ut at time t. The areas under the splash profiles are not equal to the immersed area of the wedge, except of course in the incompressible case $M = 0$. The profiles in fact illustrate how the amount of compression of the liquid builds up as the Mach number increases, since the elevations are more and more confined to the regions near the wedge. The total increases in area of the free surface are approximately 10 % less for $M = 0{\cdot}2$, 25 % less for $M = 0{\cdot}5$ and 45 % less for $M = 1$, compared with that of the corresponding incompressible case $M = 0$.

For a wedge of small but finite semi-vertex angle ϵ the maximum elevation occurs where the free surface of the liquid meets the side of the wedge. The value of x at this point is given approximately by $x = \epsilon Ut$, and expanding (23) for ϵ small it follows that the maximum free surface elevation is given by

$$\eta/Ut = -(2\epsilon/\pi)\log\epsilon + (2\log 2/M - 2 + I_0)\,\epsilon/\pi + O(\epsilon^2), \tag{24}$$

where $$I_0 = \frac{-1}{(1-M^2)^{\frac{1}{2}}}\log\left(\frac{1+(1-M^2)^{\frac{1}{2}}}{1-(1-M^2)^{\frac{1}{2}}}\right) \quad \text{if} \quad M \leqslant 1,$$

and $$I_0 = \frac{2}{(M^2-1)^{\frac{1}{2}}}\tan^{-1}(M^2-1)^{\frac{1}{2}} \quad \text{if} \quad M \geqslant 1.$$

Outside the wedge, on the liquid surface, the elevations are thus seen to be small, and this shows that the errors made in deriving the linearized boundary conditions (4) and (8) are in general of $O(\epsilon^2)$.

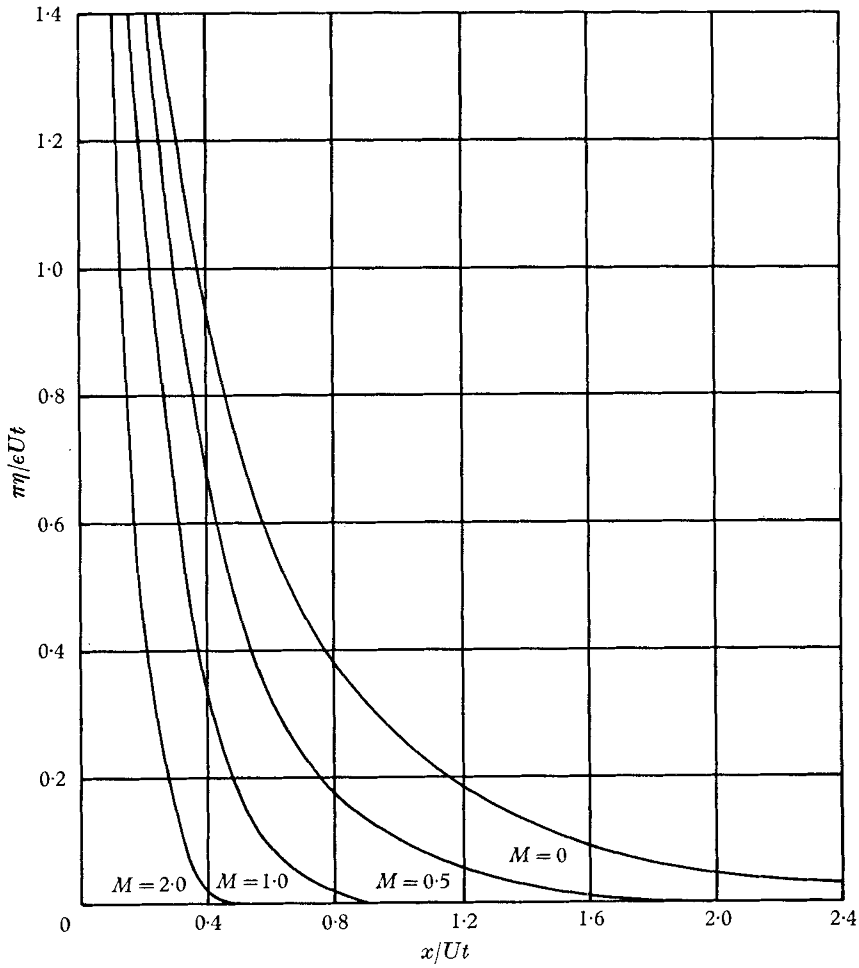

FIGURE 2. Free-surface elevations $\eta(x, t)$ for a thin wedge of semi-vertex angle ϵ at time t, for various Mach numbers M of the body's motion in the liquid.

5. The pressure distribution and drag

The pressure at different points in the liquid may be found from (7). A differentiation with respect to time then yields

$$\frac{\partial P}{\partial t} = \rho_0 \frac{\partial^2 \phi}{\partial t^2}. \tag{25}$$

Incorporating the conditions $\phi = 0$ and $\partial\phi/\partial t = 0$ at $t = 0$, the operational form of (25) may be found from the solution (15), i.e.

$$\frac{\partial \tilde{\bar{P}}}{\partial t}(p, y, aq) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{\rho_0 \epsilon M^3 a^3 q}{q^2 - M^2(q^2+p^2)} [\exp\{\sqrt{(p^2+q^2)}\, y\} - \exp(qy/M)]. \tag{26}$$

The transform (26) is now in a form which may be inverted directly by a method similar to that given in the last section. In the special case $x = 0$, the result is

$$\left.\begin{aligned}\frac{\partial P}{\partial t}(0,y,t) &= \frac{2\epsilon\rho_0 U^3 aty}{\pi(U^2t^2-y^2)(a^2t^2-y^2)^{\frac{1}{2}}} && \text{if } at > |y|,\\ &= 0 && \text{if } at < |y|.\end{aligned}\right\} \qquad (27)$$

(27) applies without loss of accuracy on the surface of the wedge, where x is $O(\epsilon)$, since the order of neglected terms is already $O(\epsilon^2)$. The pressure P on the wedge may therefore be obtained by an integration with respect to time, using the condition $P \to P_0$ as $y \to 0$ for a fixed value of t. The result may be written in terms of a pressure coefficient $C_p = (P-P_0)/\frac{1}{2}\rho_0 U^2$, and the ratio, $\beta\,(= -y/Ut)$, in the form for $M \leqslant 1$,

$$C_p = \left(\frac{2\epsilon}{\pi(1-M^2)^{\frac{1}{2}}}\right)\log\left(\frac{\{\beta^2(1-M^2)\}^{\frac{1}{2}}+\{1-M^2\beta^2\}^{\frac{1}{2}}}{\{\beta^2(1-M^2)\}^{\frac{1}{2}}-\{1-M^2\beta^2\}^{\frac{1}{2}}}\right) \quad \text{for} \quad Ut \geqslant -y \geqslant 0, \qquad (28a)$$

and for $M \geqslant 1$,

$$\left.\begin{aligned}C_p &= \left(\frac{2\epsilon}{\pi(M^2-1)^{\frac{1}{2}}}\right)\left[\tfrac{1}{2}\pi - \tan^{-1}\left\{\frac{1-M^2\beta^2}{\beta^2(M^2-1)}\right\}^{\frac{1}{2}}\right] && \text{for} \quad at \geqslant -y \geqslant 0,\\ &= \epsilon/(M^2-1)^{\frac{1}{2}} \quad \text{for} \quad Ut \geqslant -y \geqslant at. &&\end{aligned}\right\} \qquad (28b)$$

The pressure distributions for a range of Mach numbers are given in figure 3, which shows C_p/ϵ against $\beta\ (= -y/Ut)$. For $M > 1$, $C_p = \epsilon/(M^2-1)^{\frac{1}{2}} = \text{const.}$, for $Ut \geqslant -y \geqslant at$. This result is identical with that occurring in the linearized theory of a thin wedge placed in a supersonic stream of infinite extent, and is as might be expected from referring to figure 1 (*b*). This result does, however, provide a convenient check on the pressure coefficient C_p in the region $at \geqslant -y$ as it increases from zero at the free surface to $\epsilon/(M^2-1)^{\frac{1}{2}}$ when $y = -at$.

The drag coefficient $C_D = \epsilon\int_0^1 C_p\, d\beta$ is shown in figure 4 as a function of M. It is interesting to note that the maximum drag coefficient occurs at $M = 1$. An attempt has been made to compare these results with those calculated from the Prandtl–Glauert rule, shown as a broken line in figure 4. This rule applies in the case of an aerofoil moving subsonically in an infinite medium and predicts an increase in drag by a factor of $1/(1-M^2)^{\frac{1}{2}}$ due to compressibility, giving $C_D(M) = C_D(0)/(1-M^2)^{\frac{1}{2}}$. It can be seen that the actual drag increase due to compressibility is in fact very much less than would be expected using the Prandtl–Glauert rule. For example, at $M = 0{\cdot}7$ the drag coefficient is only 10 % larger than the incompressible result, while the rule quoted above predicts an increase of 40 %. It would thus appear that the free surface, in providing by its presence a surface of free movement on which the pressure is a constant, acts as a means of relieving the effects of compressibility. As a consequence, smaller pressure changes due to compressibility are caused, in comparison with those experienced in an infinite medium. This effect is also borne out by the curves of different Mach numbers shown in figure 3. The percentage pressure differences between the curves are smaller for $-y/Ut$ small, near the free surface, than those for $-y/Ut$ taking larger values, at points nearer the vertex of the wedge.

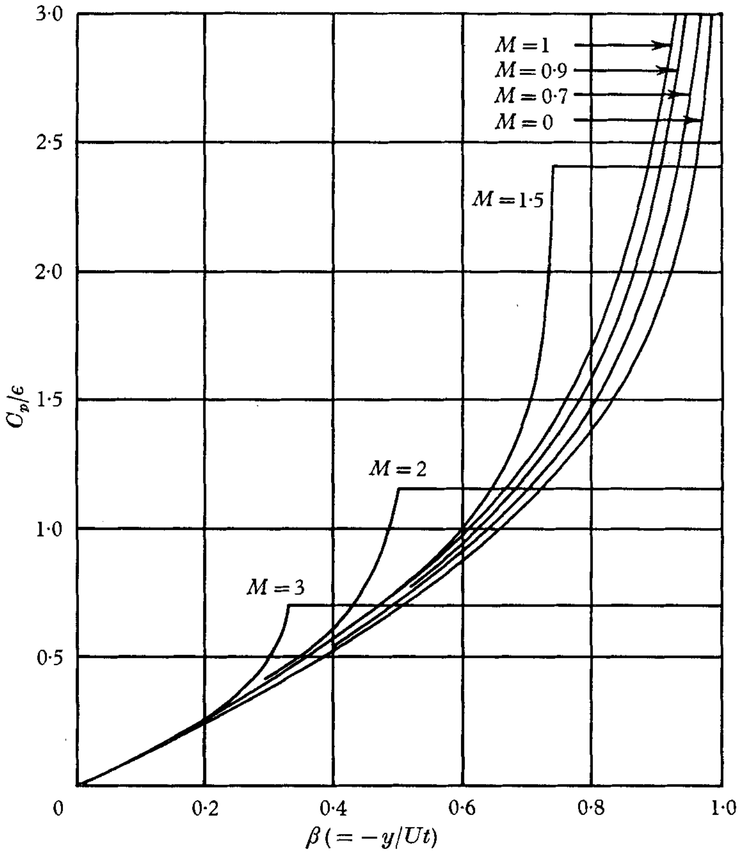

FIGURE 3. The pressure coefficient $C_p = (P-P_0)/\frac{1}{2}\rho_0 U^2$ on the wedge for various values of M.

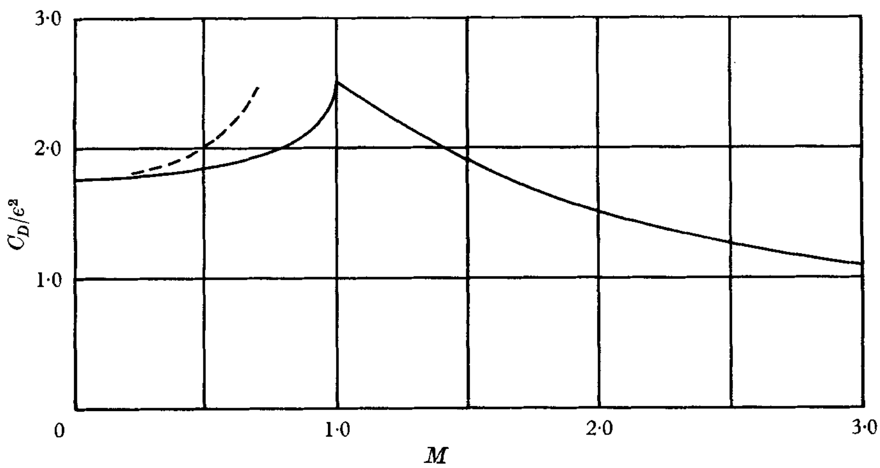

FIGURE 4. The drag coefficient $C_D = \epsilon\int_0^1 C_p d\beta$ as it varies with M. The broken line is the Prandtl–Glauert correction $C_D(0)/\epsilon^2(1-M^2)^{\frac{1}{2}}$.

6. Conclusion

The effect of compressibility has been evaluated for two important properties of the normal motion of a thin symmetric wedge into the free surface of a compressible liquid. The complete range of Mach numbers M is allowed for in the linearized theory used, and no difficulties are experienced near $M = 1$. The two properties considered are:

(i) the free-surface elevation, and

(ii) the pressure distribution and drag.

In (i), the splash profiles obtained illustrate directly the considerable area compression (per unit thickness) which is taking place. The steepening and concentrating of the profiles into regions close to the body as M increases can also be seen. In (ii), results show that the effects of compression for small M are less marked than might at first have been expected. This may be explained by the double role which the liquid–gas interface plays in providing a surface at constant pressure which is able to move freely.

Finally, it may be noted that no direct use has been made of the geometric similarity of the motion at each instant of time. Thus it would appear likely that the double-transform approach used in the present paper could be extended to deal with the normal motion of a thin symmetrical aerofoil of arbitrary shape, either finite or infinite in length.

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